

Dynamical system analysis of generalized energy-momentum-squared gravity

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In this work we have investigated the dynamics of a recent modification to the general theory of relativity, the energy-momentum squared gravity model $f(R, \mathbf{T}^2)$, where R represents the scalar curvature and \mathbf{T}^2 the square of the energy-momentum tensor. By using dynamical system analysis for various types of gravity functions $f(R, \mathbf{T}^2)$, we have studied the structure of the phase space and the physical implications of the energy-momentum squared coupling. In the first case of functional where $f(R, \mathbf{T}^2) = f_0 R^n (\mathbf{T}^2)^m$, with f_0 constant, we have shown that the phase space structure has a reduced complexity, with a high sensitivity to the values of the m and n parameters. Depending on the values of the m and n parameters, the model exhibits various cosmological epochs, corresponding to matter eras, solutions associated to an accelerated expansion, or decelerated periods. The second model studied corresponds to the $f(R, \mathbf{T}^2) = \alpha R^n + \beta (\mathbf{T}^2)^m$ form with α, β constant parameters. In this case a richer phase space structure is obtained which can recover different cosmological scenarios, associated to matter eras, de-Sitter solutions, and dark energy epochs. Hence, this model represent an interesting cosmological model which can explain the current evolution of the Universe and the emergence of the accelerated expansion as a geometrical consequence.

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I. INTRODUCTION

The discovery of the accelerated expansion of the Universe [1,2] at the turn of the last century has come as the most unexpected and surprising result for the scientific society. The reason behind the reaction is quite obvious as gravity being an attractive force will compel the Universe and all the matter present inside it to contract. So the expansion of the Universe would gradually slow down and finally reach a situation where it stops totally. After this gravity will pull it back and the Universe would undergo a contraction. But the observational evidences are saying a totally different story. This is where our standard knowledge of physics is falling short and we are compelled to

search for some new physics which will help us to explain the phenomenon that our observations are showing.

Over the last two decades, the scientific society have left no stones unturned in its quest for a suitable physical theory that will explain the accelerated expansion [3] of the Universe. The whole effort can be broadly classified into two categories. The first one targets the nature of matter that fills the Universe. This theory tells us that the Universe is filled with a mysterious negative pressure component termed as “dark energy” which provides an antigravitating stress to sustain not only an expanding Universe but also fuels it to reach a level of accelerated expansion. The most common way of doing this is by introducing a cosmological constant Λ in the Einstein’s equations of general relativity. The idea seems to be consistent but the concept is still in its infancy and is plagued by a lot of shortcomings. The most prominent one being its invisible nature and hence the term “mysterious.” Moreover there is the cosmological

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Thermodynamics in $f(R, \mathcal{L})$ theories: Apparent horizon in the FLRW spacetime

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In this paper we study the recently proposed $f(R, \mathcal{L})$ theories from a thermodynamic point of view. The uniqueness of these theories lies in the fact that the space-time curvature is coupled to the baryonic matter instead of exotic matter (in the form of scalar field). We investigate the viability of these theories from the point of view of the thermodynamic stability of the models. To be more precise here we are concerned with the thermodynamics of the apparent horizon of Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime in the background of the $f(R, \mathcal{L})$ theory. We consider several models of $f(R, \mathcal{L})$ theories where both minimal and nonminimal coupling has been considered. Various thermodynamic quantities like entropy, enthalpy, internal energy, Gibbs free energy, etc. are computed and using their allowed ranges various model parameters are constrained.

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I. INTRODUCTION

Late time accelerated expansion [1,2] of the universe is the biggest riddle of modern cosmology. Logically thinking, gravity being attractive in nature will tend to slow down the expansion of the universe in late times. But the observations are speaking a completely different story. Indeed the universe has entered into a phase of accelerated expansion and quite naturally there has been no satisfactory explanation to this phenomenon till date. Einstein's theory of general relativity (GR) is totally inconsistent with this phenomenon and we are left with no choice other than resorting to modifying the field equations of GR so that the modified equations can satisfactorily incorporate the accelerated expansion.

To date all the attempts of modifying the Einstein's field equations can be broadly classified into two categories. The first category incorporates exotic nature in the matter content of the universe, and is termed as *dark energy*. The second category modifies the gravity component of the equation, thus bringing about changes in the space-time geometry. This leads to the concept of *modified gravity theory*. Here we are interested in this second category which attempts at modifying the curvature of space-time.

The simplest model of modified gravity is the Λ CDM model where cold dark matter is coupled with the cosmological constant Λ . This cosmological constant has an antigravity effect that drives the accelerated expansion. A special class of models attempt to modify the gravitational Lagrangian in the Einstein-Hilbert action by replacing $\mathcal{L}_{GR} = R$ by an analytic function of the scalar curvature given by $\mathcal{L}_{f(R)} = f(R)$. This model helps us to explore the nonlinear effects of scalar curvature in the evolution of the universe by considering arbitrary functions of R in the gravitational Lagrangian. Extensive reviews in $f(R)$ gravity can be found in the Refs. [3,4]. Another class of models considers nonminimal coupling (NMC) between matter and curvature [5–9]. These models have been quite successful at explaining the postinflationary preheating [10] and cosmological structure formation [11–13]. Further these models have also been able to successfully mimic dark energy [14–16] and dark matter [17–19].

Most models of NMC have incorporated coupling between curvature and scalar field [20–25]. But extension of this coupling to baryonic matter content has been very rare in literature. Recently, a dynamical system analysis approach was used to analyze a model that incorporated both $f(R)$ theories and a NMC with the baryonic matter content [26]. Ref. [27] extended this coupling to the baryonic matter content and studied a more general class of $f(R, \mathcal{L})$ theories via a dynamical system analysis, where \mathcal{L} represents the matter Lagrangian. Here we are motivated to study the thermodynamical aspects of such $f(R, \mathcal{L})$ group of theories. The motivations to study such theories

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A non-static quantum inspired spacetime in $f(R)$ gravity: Gravity's rainbow

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The author dedicates this work to the memory of the people who died due to COVID-19

Abstract

In this note we explore a non-static spacetime in quantum regime in the background of $f(R)$ gravity. The time dependent Vaidya metric which represents the spacetime of a radiating body like star is studied in an energy dependent gravity's rainbow, which is a UV completion of General Relativity. In our quest we have used gravitational collapse as the main tool. The focus is to probe the nature of singularity (black hole or naked singularity) formed out of the collapsing procedure. This is achieved via a geodesic study. For our investigation we have considered two different models of $f(R)$ gravity, namely the inflationary Starobinsky's model and the power law model. Our study reveals the fact that naked singularity is as good a possibility as black hole as far as the central singularity is concerned. Via a proper fine tuning of the initial data, we may realize both black hole or naked singularity as the end state of the collapse. Thus this study is extremely important and relevant in the light of the Cosmic Censorship hypothesis. The most important result derived from the study is that gravity's rainbow increases the tendency of formation of naked singularities. We have also deduced the conditions under which the singularity will be a strong or weak curvature singularity. Finally in our quest to know more about the model we have performed a thermodynamical study. Throughout the study we have obtained results which involve deviation from the classical set-up. Such deviations are expected in a quantum evolution and can be attributed to the quantum fluctuations that our model suffers from. It is expected that this study will enhance our knowledge about quantization of gravity and subsequently about the illusive theory of quantum gravity.

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